IB Math HL, Year 1: Summer Packet

Due on the first day of class

Name: _____

Welcome to IB Math: Analysis and Approaches, higher level!

You are about to begin a rigorous, two-year course that covers topics in Algebra, Functions, Trigonometry, Geometry, Probability, Statistics, and Calculus. The first year of this course plans to cover the basics of each of those topics, so that we can dive deeper in to each topic in Year 2. In Analysis and Approaches (AA), the Year 1 course is taught similarly to an Precalculus course, with some focus on differential calculus toward the end of the year. Next year, you will take the IB Math AA HL exam, a two-year, cumulative exam. If this sounds overwhelming, stop, and take a breath–my job as your teacher is to develop your quantitative reasoning skills and give you the foundation you'll need to succeed in Year 2.

Since so much of this course relies on the concepts you have learned thus far in mathematics, this packet has been designed to help you brush up on your skills so you can hit the ground running in August. This course is rigorous and demanding, and so it is crucial that you enter the course in August with a solid foundation in these topics. You should consider working as independently as possible. However, you are also encouraged to use any online resource at your disposal to help relearn or review these skills (I especially recommend Khan Academy). If you choose to use an online resource, please indicate on which problems you used it, so I know where you had some difficulty. It's okay to get help– in fact, I expect you to in some cases– just be honest about when and where you did.

As of right now, I'm supposed to teach this class in the fall, and **I plan to count this assignment as a series of homework grades. There will also be a quiz on this content within the first few days of class.** I'm using this packet both for you to practice your math skills and for me to figure out any problem areas I should address right away. We will not have extensive time to backtrack on this content, but I will still use this as an important tool in structuring Year 1 of this course.

You must show all work to receive full credit, and work should be completed neatly and thoroughly, preferably in pencil. In the interest of saving some paper, I didn't provide a lot of room to complete these problems, so **please work on separate sheets of paper, and attach them to this packet before submitting.**

If you have any questions at all over the summer, please reach out to me! My email is **rcox@theproutschool.org**. I would be happy to Zoom with you on a case-by-case basis if you're having difficulty.

Have a great summer!

- Mr. Cox

A brief introduction to IB notations and commands

Notation

Number sets	$\mathbb{N}$	The set of positive integers and zero (natural numbers), $\{0, 1, 2, 3, \dots\}$
	$\mathbb{Z}$	The set of integers, $\{0, \pm1, \pm2, \pm3, \dots\}$
	$\mathbb{Z}^+$	The set of positive integers, $\{1, 2, 3, \dots\}$
	$\mathbb{Q}$	The set of rational numbers, any number that can be written as a fraction in simplest form
	$\mathbb{Q}^+$	The set of positive rational numbers, $\{x \mid x \in \mathbb{Q}, x > 0\}$
	$\mathbb{R}$	The set of real numbers
	$\mathbb{R}^+$	The set of positive real numbers, $\{x \mid x \in \mathbb{R}, x > 0\}$
Absolute value	$\lvert x \rvert$	IB may refer to this as *modulus*
Line segments		Line segments $\overline{AB}$ will be written as $[AB]$
Angles		We typically write angle A as $\angle A$. IB will use the notation $\hat{A}$ or $C\hat{A}B$
Repeating decimals		Standard notations: $0.\overline{3} = 0.3333\dots$, $0.\overline{123} = 0.123123\dots$ IB notation: $0.\dot{3}$, $0.\dot{1}2\dot{3}$
Slope		IB will refer to this as the *gradient*
Graphing calculator		IB will refer to this as a *GDC* (graphic display calculator). The TI-83 Plus/TI-84 Plus, as well as similar Casio models, are recommended. The TI-Nspire is prohibited for IB because of the computer algebra system (CAS) installed.

Key Command Terms

Draw	Represent by means of a labeled, accurate diagram or graph, using a pencil. A ruler should be used for straight lines. Diagrams should be draw to scale. Graphs should have points correctly plotted and joined in a straight line or curve.
Hence	Use the preceding work to obtain the required result.
Hence or otherwise	It is suggested that the preceding work is used, but other methods could also receive credit.
Show that	Obtain the required result (possible using the information given) without the formality of proof. These questions do not generally require the use of a calculator.
Sketch	Represent by means of a diagram of graph (labelled as appropriate). The sketch should give a general idea of the required shape or relationship, and should include relevant features.
Write down	Obtain the answer(s), usually by extracting information. Little to no calculation is required. Working does not need to be shown.

1 Functions

1.1 Quadratic polynomials and equations

1-6. Factor each expression completely.

1. $x^2 - x - 2$
2. $x^2 + 3x - 4$
3. $16x^2 - 81y^2$
4. $3x^2 - 5x + 2$
5. $2x^2 - x - 6$
6. $x^3 - 3x^2 - 18x$

7-12. Solve each equation by using any method (factoring, completing the square, or the quadratic formula). You may not solve by graphing or by guess-and-check. Give exact answers as solutions.

7. $x^2 + 25 = 10x$
8. $x^2 + 3x - 1 = 0$
9. $x + \dfrac{12}{x} = 7$
10. $x^2 + 2 = 9$
11. $x^2 - 5x = 0$
12. $36x^2 - 35 = 0$

1.2 Quadratic functions

13-15. For each of the following, state the axis of symmetry, vertex, concavity, x-intercept(s), and y-intercept(s). Then, sketch the graph of each function, clearly labeling the intercepts and vertex.

13. $y = -2(x + 2)(x - 1)$
14. $y = \dfrac{1}{2}(x - 2)^2 - 4$
15. $y = 2x^2 + 6x - 3$

16-20. Use your graphing calculator[1] for the following questions. Round all approximations to three significant figures.

16. Find the roots of $3x^2 - x - 5 = 0$.

17. Find the minimum value of $f(x) = 2x^2 - 5x + 1$.

18. Find the maximum value of $g(x) = -3x^2 + x - 3$.

19. Find the points of intersection of $y_1 = 3 - 5x - x^2$ and $y_2 = x^2 + 3x + 11$.

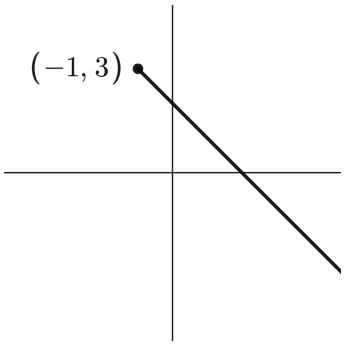
20. Find the points of intersection of $y_1 = x^2 + 3x - 1$ and $y_2 = 5 - x$.

[1] IB will refer to your graphing calculator as a *graphic display calculator*, or GDC.

1.3 Domain and range

21-26. State the domain and range of each function without the use of a calculator. Express your answers using interval notation. If there is not a point clearly marking the end of the curve, assume that the curve continues infinitely in that direction.

21.



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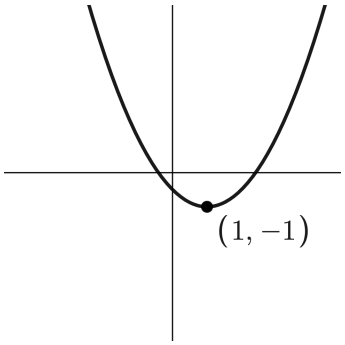

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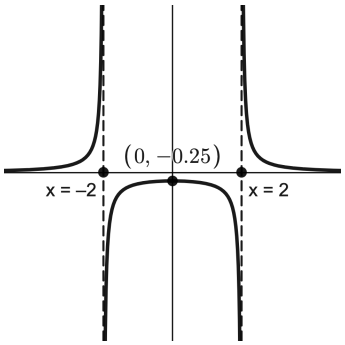


24. $f(x) = \sqrt{x-4}$

25. $g(x) = 5x - 3x^2$

26. $y = \dfrac{x+4}{x-2}$

1.4 Operations between functions

27-32. Let $f(x) = 2x^2 - 1,\ g(x) = 3x,\ h(x) = 5 - x$. Find each of the following.

27. $f(-3)$

28. $(f \circ g)(x)$

29. $(h \circ f)(x)$

30. $(f \circ h)(x+1)$

31. $(g \circ h)(4)$

32. $(f \circ f)(-1)$.

33-36. Find the inverse of each of the following.

33. $f(x) = 2x + 1$

34. $f(x) = \dfrac{x^3}{3}$

35. $g(x) = \dfrac{5}{x-2}$

36. $g(x) = 1 + \sqrt{4-x}$

37. If the point $(2, 7)$ is on the graph of $f(x)$, what point must lie on the graph of $f^{-1}(x)$?

1.5 Linear functions

38. Write down the slope, x-intercept, and y-intercept of the equation $5x - 4y = 8$.

39. Determine the equation of the line in slope-intercept form that passes through the points $(4, 3)$ and $(7, -2)$.

40. Determine the equation of the line that passes through the point $(4, 7)$ and is perpendicular to the line $y = -2x + 9$.

1.6 Rational functions

41-43. Find the equations of the horizontal and vertical asymptotes of each of the following.

41. $y = \dfrac{1}{2x - 5}$

42. $y = \dfrac{x^2 - 5}{2x^2 - 12}$

43. $y = \dfrac{x^2 + 2x - 3}{x^3 + 6x^2 - 7x}$

1.7 Parent functions and basic transformations

44-48. Graph the parent function f and the transformation g on the same set of axes for each pair of functions. Label all key features of g (maxima, minima, intercepts).

44. $\begin{cases} f(x) = x \\ g(x) = 3x - 4 \end{cases}$

45. $\begin{cases} f(x) = x^2 \\ g(x) = x^2 - 5 \end{cases}$

46. $\begin{cases} f(x) = x^3 \\ g(x) = x^3 + 1 \end{cases}$

47. $\begin{cases} f(x) = \sqrt{x} \\ g(x) = \sqrt{x + 6} \end{cases}$

48. $\begin{cases} f(x) = |x| \\ g(x) = |x - 1| + 3 \end{cases}$

2 Number and algebra

2.1 Solving equations

49-53. Solve for x. Eliminate any extraneous solutions, if necessary.

49. $\sqrt{37 - 3x} = x - 3$

50. $-3(2x + 1)^3 = -192$

51. $\dfrac{x}{3} - \dfrac{5}{2} = \dfrac{-3}{x}$

52. $\dfrac{4x - 1}{x + 1} = x - 1$

53. $2|3x - 1| + 5 = -2x + 8$

2.2 Solving inequalities

54-57. Solve each inequality. Express your answers in interval notation.

54. $5(x-3) \leq 8(x+5)$

55. $3(6x-1) > 18-3x$

56 $26+m \geq 5(-6+3m)$

57. $-2(1-5x) > -(x+1)-1$

2.3 Solving systems of equations

58-61. Solve algebraically (without a graphing calculator).

58. $\begin{cases} 3x+7y=36 \\ x=5y-10 \end{cases}$

59. $\begin{cases} 6x+10y=32 \\ 4x-2y=4 \end{cases}$

60. $\begin{cases} x=y^2 \\ x-y=6 \end{cases}$

61. $\begin{cases} x^2+y^2=25 \\ y=x^2-13 \end{cases}$

62. On a graph, where does the solution(s) to a system of equations lie?

2.4 Simplifying expressions

63-68. Simplify each radical expression, rationalizing the denominator where necessary. Provide exact answers.

63. $\sqrt{72}$

64. $\dfrac{3}{\sqrt{5}}$

65. $\dfrac{1}{\sqrt{x}}$

66. $\dfrac{1}{1+\sqrt{3}}$

67. $\dfrac{1}{\sqrt{x}-2}$

68. $\sqrt{48x^6}$

69-71. Simplify without the use of a calculator.

69. $8^{2/3}$

70. $81^{-3/4}$

71. $(9x^2)^{1/2}$

72-77. Simplify completely, leaving only positive exponents in your answer.

72. $(x^2y)^4$

73. $\dfrac{3x^2y^{-3}}{12x^6y^3}$

74. $(5x^2y)(2x^4y^{-3})$

75. $\left(\dfrac{4x^5y}{16xy^4}\right)^3$

76. $\dfrac{2x^4y^{-4}}{8x^7y^3}$

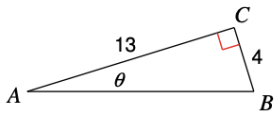
77. $(2x^3y^{-3})^2$

3 Geometry and Trigonometry

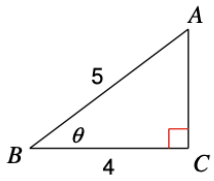
3.1 Right triangle trigonometry

78-83. Use the Pythagorean Theorem to find the missing side of the right triangle. Then, find the exact value of the indicated trigonometric ratio.

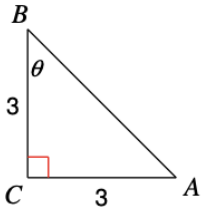
78. $\sin\theta$



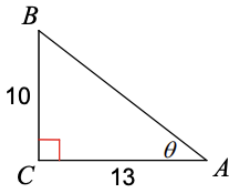
79. $\cos\theta$



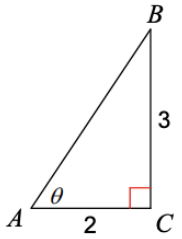
80. $\tan\theta$



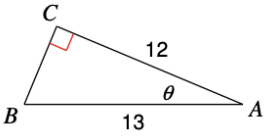
81. $\sin\theta$



82. $\cos\theta$

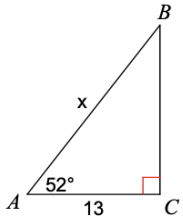


83. $\tan\theta$



84-85. Use a trigonometric ratio to find the measure of the indicated side. Round your answers to three significant figures.

84.



85.